Topic Study Group 11

Teaching and Learning of Probability

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ABSTRACT An overview of the papers that were submitted and accepted to TSG-11 is presented. First, we present the document "Purposes and subthemes" with which we call for papers. Second, we make a very brief description of some features of the papers, organized by school level (primary, secondary and tertiary) and each of these divided according to whether they refer to students or teachers. Next, some relationships between the subthemes of the initial document and the topics that are actually present in the papers are pointed out. Finally, some recommendations are formulated.

Keywords: Probability; Probability education; Probabilistic thinking.

1. Purpose and Subthemes

The general aim of the Topic Study Group on Teaching and Learning of Probability (TSG-11) was to continue the relatively recent, albeit ever-growing trend of providing a dedicated venue to promote the discussion of a variety of perspectives related to probabilistic thinking and the learning and teaching of probability. TSG-11 at the 14th International Congress on Mathematical Education (ICME-14) attempted to provide an overview of the international discussion on probability education, as broadly as possible, by building upon the more recent literature from the field. Further, TSG-11 made every effort to display the progress of the discussion in the intervening years since ICME-13 and ICME-12. Lastly, we would be remiss not to mention that we, to the best of our ability, allowed for insight into less well-known strands of the discussion from researchers around the world, especially those from underrepresented countries. To meet these general and specific objectives, we identified five subthemes for TSG-11.

- *Conceptual frameworks to develop probabilistic thinking.* To continue the emerging creation of frameworks to describe or model the development and growth of probabilistic thinking of students especially at intermediate and tertiary levels. We recognized the importance of discussing models of

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students' process of integrating the different philosophical interpretations (e.g., classical, frequentist, subjective, and others) of probability.

- *Connecting probability with statistics.* The development of probabilistic notions through experiments, data explorations and simulations can help students to build basic connections between statistics and probability, but it is required to understand how the process of concept formation emerges in students under such conditions.
- *The role of technology in teaching and learning probability.* The availability of increasingly powerful technology and software for statistical and probabilistic education requires, in addition to the inherent innovation, a parallel development of the theoretical reflection and conceptualization of empirical experiences.
- *Task design and learning trajectories.* One way to ensure that the knowledge accumulated by research in education in probability develops into educational practices is through the design of tasks and learning trajectories to promote the thinking and reasoning of integrated probabilistic concepts, including modeling processes.
- *Probabilistic knowledge for teaching.* Understanding and deepening the knowledge that teachers need to teach probability can help solve potential problems with their learning so that they provide a comprehensive education that includes probability. The availability of models that describe and conceptualize the probabilistic knowledge of teachers and their relationship with their teaching practice is important.

We, of course, welcomed submissions that fell outside the presented topics but within the teaching and learning of probability.

2. General Organization of Papers Presented

TSG-11 at ICME-14 had 21 presentations: three invited lectures of 20 minutes each, seven "long" presentations of 15 minutes and eleven "short presentations" of 10 minutes (Tab. 1 on the next page). Invited lecturers were: Amy Renelle, Stephanie Budgett and Rhys Jones who presented "A consideration of alternative sample spaces used in coin-toss problems"[10]; Vincent Martin, Mathieu Thibault and Marianne Homier presented "Self-reported practices of probability teaching: the use of the frequentist approach, manipulatives and technological tools"[21]; and, Gale Russell who presented "From towers of linking cubes to the binomial expansion theorem: what can be learned about combinatorics?"[18]. In what follows, we summarize the distribution of all 21 papers across three variables: Students/Teacher, School level (Primary, Secondary, Tertiary), and the use (or not) of Digital technology.

In addition to the first fifteen papers[1–15] on studies with students across different ages, the other six papers[16–21] involved pre- or in-service teachers. Parsing a bit further, that is, considering the school level at which the study is focused, the first 15 are distributed as follows: Two papers[1,2] refer to primary school students, nine[3–9; 13,14] to secondary school (middle and high school) and four[10–12;15] at the tertiary level. Of

the remaining six, two papers[16,20] do refer to primary school teachers, three[17–19] to secondary school teachers, and one[21] to both primary and secondary school teachers. Five papers included technological resources in their research, two[13,14] were with high school students, one[15] with university students. Of the remaining two, one paper[20] involved pre-service teachers and the other[21] with primary and secondary in-service teachers. We hope that this context, presented in this manner, will help all interested readers better navigate our list of conference papers.

Tab. 1 Papers presented in TSG-11

	Paper and author(s)
[1]	The emerging interplay between subjective and objective notions of probability in young children. ***Sibel Kazak*** (Turkey) *and Aisling Leavy* (Ireland)
[2]	Children's apatial cognitive strategies and their development from the perspective of microgenesis. *Zikun Gong and* ***Du Zhang*** (China).
[3]	Developing a learning progression for probability based on the GDINA model in China. ***Shengnan Bai****, Jiwei Han, Kaijun Zhang, and Xueming Gao* (China).
[4]	How can probability reasoning protect adolescents from problem gambling? ***Catterine Primi*** *and Maria Anna Donati* (Italy).
[5]	Confidence and competence of Indonesian secondary school students in completing probability tasks: findings from a pilot study. ***Bustang Bustang*** (UK).
[6]	Problem sequences for developing two basic notions: probability and expected value in Hungarian secondary schools. ***Ôedoen Vancsó*** *and Eszter Varga* (Hungary).
[7]	The frequentist approach of probability, from random experiment to sampling fluctuation. ***Jannick Trunkenwald*** (France)*, Fernand Malonga-Moungabio* (Congo)*, and Dominique Laval* (France).
[8]	Secondary school students' strategies in solving permutation problems. ***Luca Lamanna*** (Italy)*, Magdalena M. Gea-Serrano* (Spain)*, and Carmen Batanero* (Spain).
[9]	Establishing connections between language and probabilistic notions through a wodb task. ***Maria Ricart****, Pablo Beltrán-Pellicer, and Assumpta Estrada* (Spain).
[10]	A consideration of alternative sample spaces used in coin-toss problems. ***Amy Renelle****, Stephanie Budgett, and Rhys Jones* (New Zealand).
[11]	Is it in the cards? Revealing consequential probability. ***Egan J. Chernoff****, Nat Banting, and Ryan Banow* (Canada).
[12]	Use of the empirical rule in the course of probability: an application proposed by students. ***Beatriz A. Rodríguez González****, Omar Alejandro Guirette Barbosa, Gabriela Noemi Figueroa Ibarra, Hector Antonio Durán Muñoz* (Mexico)*, and Difariney González Gómez* (Colombia).
[13]	High-school students' probabilistic reasoning when working with random intervals. *Sandra A. Martínez Pérez and* ***Ernesto Sánchez*** (Mexico).
[14]	The computer simulation as a resource to teach normal distribution. ***Jesús Salinas*** *and Julio César Valdez* (Mexico).
[15]	Modeling eliciting activities for the teaching of the probability in a computational environment. ***Sandiago Inzunza*** (Mexico).
[16]	Alice in randomland: differences in attitudes of future primary school teachers towards probability and its teaching. *Claudia Vásquez,* ***Flavio Guiñez****, Camila Brito, and Salomé Martínez* (Chile).
[17]	Teachers' epistemological assumptions that tend to govern their pedagogy while teaching probability. ***Haneet Gandhi*** (India).
[18]	Concretely developing the binomial expansion theorem: where did the permutations go? ***Gale Russell*** (Canada).
[19]	The mathematical work of secondary teachers in the domain of probability in Chile. ***Katherine Machuca Pérez*** (Chile).
[20]	Understanding elements of a randomization test. ***Susanne Podworny*** (Germany).
[21]	Self-reported practices of probability teaching: the use of manipulatives and technological tools. ***Vincent Martin***, ***Mathieu Thibault****, and* ***Marianne Homier*** (Canada)

3. Brief Indications on the Topics of the Papers

Regarding the research with primary students, Kazak and Leavy[1] focused on the children's estimations of the likelihood of outcomes from chance experiments observing the interplay between subjective and objective notions of probability, and Gong and Zhang[2] addressed emerging cognitive strategies when children face sample space tasks, as well as how and how quickly they develop. The studies with pre- or in-service primary teachers focused on different topics: Vásquez et al.[16] on attitudes towards probability and its teaching; Martin et al.[21] on the self-reported practices of how teachers use the frequentist approach to probability, manipulatives, and technology in their teaching of probability; and Podworny[20] on the understanding and difficulties about the elements of randomization test. We would also note that these last two do include technology, one asking teachers how they use it and the other using computer simulations.

Considering the secondary level, Bai et al.[3] focused on developing a learning progression of probability for 7–11th grade students by using a diagnosis test of 26 items administered to 1490 Chinese students. Primi and Donati[4] reported on developing and evaluation of a school-based preventive intervention aimed to modify gambling-related distortions on at-risk adolescents, focusing the training activity on the concept of probability. Bustang[5] investigated the confidence and competence of Indonesian high school students and wonders if the biases and misconceptions that affect Western students are also present in other cultures or if the culture of the students affects their probabilistic reasoning. Vancsó and Varga[6] proposed a sequence of problems in the context of betting to develop the secondary students' notions of probability and expected value. Trunkenwald[7] pondered on students' understanding of the frequentist approach to probability, particularly, the relationship of the empirical observation of frequencies fluctuation with the idea of measuring a probability. Lamanna et al.[8] explored the effect of instruction in combinatorial reasoning of secondary school students in Italy by analyzing the students' strategies in solving two permutation problems with and without instruction. Ricart et al.[9] used the technique "Which One Doesn't Belong? (WODB)" to explore, through the mathematical vocabulary used, the ideas that students at different educational levels have about probabilistic notions. Martínez and Sanchez[13] reported on a design experiment to introduce the concept of random intervals from a frequentist approach with the aid of a technology tool and observe, in this context, the students' reasoning for making sense to frequentist approach of probability. Their paper concerned the potential of the use of technology to explore the high school students' reasoning when using the software to understand the normal distribution.

Considering teachers at high school, Gandhi[17] raised questions about the epistemological assumptions with which high school teachers approach the curricular material in their probability classes and the way in which epistemological approaches to probability become part of their pedagogy. The paper explains the evolution of an activity, carried out with pre-service teachers, consisting of starting from towers of linked cubes to arrive at the binomial expansion theorem and what can be learned with

it about combinatorics. Machuca[19] presented the design of research project for secondary teachers in which mathematic activity in front of probability modelling tasks will be developed and she wonders about the features of teachers' work when they solve the tasks.

Regarding tertiary level students, Renelle et al.[10] told us about alternative sample spaces used by participants in coin-toss problems and the paper is a reflection on whether one sequence could be more likely depending on the interpretation of the question. Chernoff at al.[11] recounted how different ways of analyzing a problem by students, leads to explore and quarrel the probabilities stemming from a simple standard deck of cards sitting on a table. Rodríguez et al.[12] proposed a perspective on problem posing in probability and statistics that involves the topics of experimental probability, the empirical rule and hypothesis testing with the aim of developing creativity and learning skills of students. And Salinas and Valdez[14] developed modelling eliciting activities with their students and asks: What challenges university students face with a model and computer simulation approach? How is their reasoning when they interact and build statistical models?

Having presented brief indications on the topics of the papers, we now switch our focus. Relationships that emerged with initial subthemes are now presented.

4. Some Relationships with Initial Subthemes

Conceptual frameworks to develop probabilistic thinking. All the papers are based on some theoretical considerations, but only a few[13–15,19] include a section of theoretical or conceptual framework. Gong and Zhang[2] built a four-level hierarchy of children's cognitive strategies, whereas others mention some conceptual framework in the introduction section or in the method. Two papers[19,14] stand out, in that the former mentions the Mathematical Working Space (MWS), and the latter addresses a documentational approach to didactics.

Connecting probability with statistics. Four papers address topics related to statistics: Podworny[20] carried out activities to understand the randomization test technique; Rodríguez et al.[12] reported on the use of the empirical rule in a probability course; in the presentation of Martínez and Sanchez[13] the students solved a problem of random intervals that later can be related to confidence intervals; and Salinas and Valdez[14] showed how to approximate the normal distribution with the help of technology.

*The role of technology in teaching and learning probability***.** Six papers included, in the investigations they report, some use of technology, but only in the case of paper[14] a question is asked about the role it plays in learning; in the other cases it has an ancillary function. Three papers[1,15,20] utilize *TinkerPlots* and two papers[13,14] use *Fathom*. Of note, a paper[21] is included, albeit indirectly because the paper asks teachers about how they use technology in teaching.

Task design and learning trajectories. Vancsó and E. Varga[6] proposed a series of betting problems to promote the understanding of probability and expected value. Podworny[20] mentioned a learning trajectory for inferential reasoning with randomization

tests. Others use problems in their investigations. For example, Kazak and Leavy[1] proposed a task to subjectively evaluate the probabilities, Ricart et al.[9] explored students' probabilistic language using the WODB technique, and Chernoff et al.[11] promoted a quarrel about the probabilities stemming from a deck of cards problem.

Probabilistic knowledge for teaching. In six papers the object of study was the probabilistic knowledge of teachers. Martin et al.[21] studied the self-reported practices of teaching by primary and secondary teachers. Gandhi[17] explored the assumptions that teachers adopt in their classes on the curricular material and the role of the three epistemological approaches of probability in their pedagogy. Vásquez et al.[16] examined the attitudes of primary teachers towards probability and its teaching. The other three focus more on teachers' probabilistic content knowledge: combinatorics and binomial expansion[18], randomization test[20], and modelling[19].

Other related notions. Several topics other than those stated in the purposes and subthemes were addressed in the papers, of which we can highlight attitudes and values[2,4,16], problem solving[6,10,11] and combinatorics[8,18].

5. Looking Ahead

Recognizing the dissonance associated with the leaders of a Topic Study Group entitled Teaching and Learning of "Probability" (which is colloquially, albeit widely known as how likely something is to happen) attempting to peer into the future, we nevertheless wish to end this proceedings report with what, we see, comment on a few possible research directions stemming from the papers presented in TSG-11 at ICME-14.

First, we contend that the use of technology in supporting learning and teaching of probability appears to shift the attention for advanced probabilistic concepts, relationships, and procedures. More research on this aspect of technology is needed to develop effective ways of promoting students' conceptual understanding. Second, while most papers addressed student learning at different educational levels, studies involving pre-service and in-service teachers were limited. With the availability of educational technology tools to support student learning, we note that there is need for further research on teacher knowledge and practices on the use of technology in their teaching and learning of probability. Third, a modeling approach to probability, especially with the use of computer simulations, is an emerging area of research. More interest in researching the modeling approach in teaching and learning of probability with teachers and students are welcome. Lastly, we wish to underscore the continual untethering of probability education from that of statistics education. Perhaps particularly present at ICME-14 due to dedicated working groups for Teaching and Learning of Statistics (TSG-12) *and* Teaching and Learning of Probability (TSG-11), we hope to see, again, a Topic Study Group dedicated specifically to probability education at the 15th International Congress on Mathematics Education in 2024 in Sydney, Australia.